TRACTABILITY

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Summary

Turing's frontier

Computability *in principle*

Algorithmic computability	ω-logic, etc

Absoluteness of the frontier (Gödel's "miracle")

- Robustness: variants of Turing machines (several tapes, etc) lead to the same notion of computability
- Invariance: other attempts to draw the frontier (λ-definability, equational systems, etc) lead to the same
- ...

In search of computability *in practice* (tractability)

1. Why and where ?

- Computation by algorithm is idealized (no limit or space ressources)
- Warranted speed is practically needed (e.g. to compute the correct position of the intrados before plane landing)

NB. Clearly, one has to look left to the frontier (*ok, you did that practically, but was that feasible in principle* ???)

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2. Soritic issue

- **Naive approach.** Specification of an explicit bound for the number of steps (let's say: P is tractable iff an algorithm provides the result in less than N steps)
- **Objection.** If n is acceptable, $n+1$ is certainly acceptable as well
- **Not insurmontable.** Cf R. Parikh, *Existence and Feasibility in Arithmetic*, JSL, XXXVI-3, 1971, 494-508
  - Consider $PAF = PA + \{\ Acceptable(0),\ Acceptable(n) \implies Acceptable(n+1),\ \neg Acceptable(N)\ \}$
  - Parikh's result: PAF is consistent if we limit to the theorems provable in less than N steps !

In search of computability *in practice* (tractability)

3. Caution

- Some respectable principles are to be avoided. E.g. **Pure** mathematics (in the sense of Hilbert's *Methodenreinheit*) are certainly not tractable: a 3-line impure proof of a simple theorem of arithmetic requires, when purified, as many lines as the number of nano-seconds since the *Big Bang* (G. Boolos, *Don't Eliminate Cut*, in *Logic, Logic, and Logic*, Harvard UP, 1999)
- **Conclusion.** Those who put tractability in a central place should be ready to renounce to some part of the most venerable heritage in philosophy of science

Offers on the market

- **Prominent proposal**: tractability amounts to algorithmic computability in polynomial time (**P**)
- **Objections.**
  1. **Over-coverture.** Some algorithms in **P** are (intuitively) unfeasible. Searching a phone number in an unsorted list of Paris subscribers is polynomial, even very simply ($P(x) = x$), but repulsive. Only the search in a sorted list (complexity log_2) is suitable
  2. **Lack of machine-invariance.** Some problems, as factorization, which are exp-time on classical computers become **P** on quantum computers (Shor's algorithm)
  3. **P** $\neq$ **NP** is still open !

A new frontier in Turing's land

1.Rationale

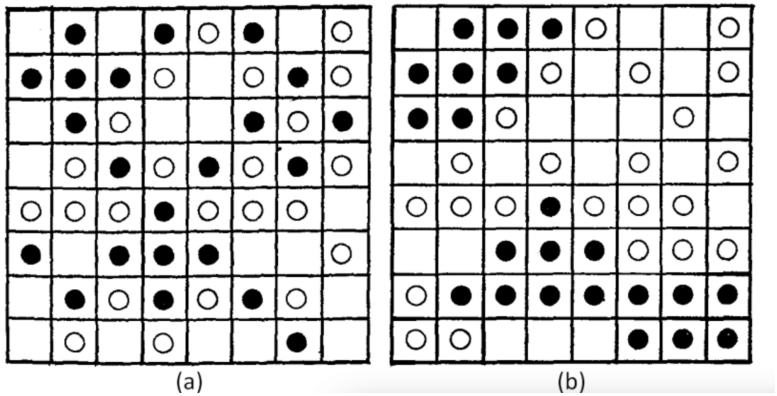
Caveat. The new frontier is not clearly visible in Turing's original format, one has to move to another format (which has been painfully proven equivalent to Turing's one), that of cellular automata

This frontier contrasts

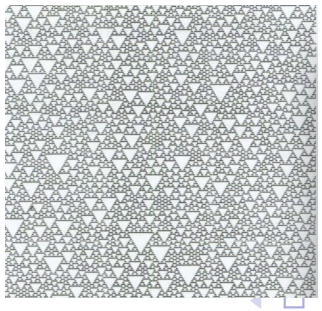
1. Situations for which the computation allow a "direct jump" from the initial to the stationary (or to a arbitrarily distant) state
2. Situations in which the best we can do is to follow step by step iterations to get the final state from the initial configuration

Contrast between

1. T.C. Schelling (1971), "Dynamic Models of Segregation", *Journal of Mathematical Sociology*, *Journal of Mathematical Sociology*, I-2, 143-186



2. Wolfram's 126th automaton starting from disorder after 300 iterations



A new frontier in Turing's land

Paternity

1. **Father**: M. Bedau, "Weak Emergence", *Philosophical Perspectives*, XI-1997, 375-399

   "A property of S (...) is "weakly emergent" iff S's possessing that property can be derived only by step by step simulation of S"

2. **Grandfather**: Leibniz, *Discours de métaphysique* (1686)

   Aim: to make the thesis of the analyticity of all the truths (included empirical ones) with the obvious lack of human omniscience.
   While all the properties of Adam (and of his descendants) are included (*inesse*) in Adam's notion, we have to wait for their occurrence to know them and we cannot do better that following step by step the chronological unfolding of this notion

A new frontier in Turing's land

Note on predictability

The current notion of predictability (ϕ is predictable iff one can know, before ϕ occurs, that ϕ will occur) is too context-dependent to do the job: meteorological forescast to 2 days, inaccessible at the time of Leibniz, is trivial today)

Thus, let's say that ϕ is *weakly predictable* iff simulating (i.e. mimicking a process P by a process P' that follows, while at another space and time scale, the successive stages of P) is the only mean to get the knowledge of the future occurrence of ϕ

The frontier is now, among predictable events, between strongly and weakly predictable ones

How to transcribe this frontier in Turingian terms of algorithmicity ?

A new frontier in Turing's land

Back to Turing's land

Let's say that a recursive function $f : \mathbb{N} \longrightarrow \mathbb{N}$ is *weakly computable* iff $f(n)$ cannot be computed before computing $f(0)$, $f(1)$, ... $f(n-1)$

The Holy Graal is the separation between strongly and weakly computable functions

1. *WCF* is inhabited, e.g. by the function $n \rightarrow n^{th} digitof\, \pi$ (think of Archimedes' exhaustion method)
2. The separation has the desired properties of absoluteness (work by Delahaye and Zwirn)
3. Conjecture (Moschovakis): $WCF = NSPACE$

Bibliography

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H. Zwirn, "Computational Irreducibility and Computational Analogy", *Complex Systems* XXIV-2, 2013

H. Zwirn & J.P. Delahaye, "Unpredictability and Computational Irreducibility", in H. Zenil (ed.), *Irreducibility and Computational Equivalence: 10 Years After Wolfram's A New Kind of Science*, Springer, Heidelberg, 2012